Is Codman's paradox really a paradox?

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Abstract

The phenomenon of Codman's paradox has been described and discussed in context of human shoulder motion. We focus on Codman's rotations from mathematical point of view and we show there is no paradox about them. The behavior of the arm is easy to explain by mathematical mechanism of 3D rotation composition. The most important thing is the nature of third Codman's rotation that is dependent on the first two. There is a possibility that the correspondence between axial rotation produced by sequence of Codman's rotations and the second Codman's rotation (*swing*) could be used in shoulder mobility diagnostics, although careful research into measuring procedures would be strongly advised.

Key words: Codman's paradox, shoulder, 3D rotation

1 Introduction

Shoulder motion is a frequently studied topic. The structure of the shoulder allows great mobility while maintaining a significant stability and support. To meet such requirements it is quite a complex system and therefore it is not easy to model. The level of studied detail is always an important factor when dealing with shoulder motion. Often the shoulder motions are being expressed it terms of arm rotations (e.g. elevation, horizontal rotation), with little respect to detailed structure consisting of four mobile connections (sternoclavicular, acromioclavicular, scapulothoracic and glenohumeral).

A phenomenon called Codman's paradox has long been described in context of shoulder (arm) motion study and has again been discussed Cheng (2006). We want to dedicate this paper to Codman's paradox. First, in section 2, we want to focus on some mathematics behind the 3D rotations and their compositions. Thus we hope to show that there is no paradox about Codman's rotations and the phenomenon is not merely a property of human shoulder motion.

Furthermore, in section 3, we will discuss possible application and usage of knowledge based on understanding Codman's paradox and real shoulder anatomy.

2 Codman's Paradox

The Codman's paradox refers to a specific pattern of motion at the shoulder joint Cheng (2006). If the arm performs three consecutive rotations that result in a closed loop motion of the arm's end, its axial rotation will be different from the axial rotation in the original posture. This difference in axial rotation corresponds with the second rotation of the sequence.

The rotations of the sequence have been called *elevation*, *swing* and *descending*. There have been certain constrains imposed on them. The first rotation must take place about the axis perpendicular to long axis of the arm. The axis of *swing* then must coincide with original long axis of the arm. *Descending* just returns the end of arm to its initial location. This is how the situation of Codman's paradox has been specified.

2.1 Mathematics of Codman's Rotations

First we should explain the basics of geometric transforms (with focus on rotations) in 3D. We will be brief as a detailed studies are available in many computer graphics textbooks (such as Eberly (2001)) or even Internet sites (Mathworld (www))

The cartesian coordinates $[x_0, y_0, z_0]$ of point P_0 are transformed to coordinates $[x_1, y_1, z_1]$ of point P_1 by geometric transformation A_1 as is shown by equation 1.

$$P_1 = A_1 \cdot P_0 \tag{1}$$

In the same way can the point P_1 be transformed to point P_2 by geometric transform A_2. If we substitute for P_1 the expression 1, we get the following relationship.

$$P_2 = A_2 \cdot A_1 \cdot P_0 = A_{21} \cdot P_0 \tag{2}$$

Equation 2 clearly shows us that the geometric transformations can be composed. The method of this composition depends on mathematic implementation of transformation. Most general implementation that can cover all linear and nonlinear transformations is transformation matrix. Transformations are composed by matrix multiplication, which is well known to be non-commutative, thus reflecting the fact that the sequence of transformations does matter. Note in equation 2 that new transformation is added to the left. The matrix is multiplied by the new transformation matrix from the left if this new transformation occurs after the others.

The three Codman's rotations – let us denote them R_I (elevation), R_{II} (swing) and R_{III} (descending) – must cause a closed loop motion of the tip of rotated object. This imposes limitations on the third rotation (R_{III}). The purpose of this rotation is to bring the tip to the original position.

The rotation R_I occurs first, then R_{II} and last is R_{III}. Our composed rotation will then be $R_{III} \cdot R_{II} \cdot R_I$ and its effect on any point P will be

$$P' = R_{III} \cdot R_{II} \cdot R_I \cdot P \quad (3)$$

which has been observed to be identical to

$$P' = R_{II} \cdot P \quad (4)$$

therefore the following expression should be true

$$R_{III} \cdot R_{II} \cdot R_I = R_{II} \quad (5)$$

This identity suggested by expressions 3, 4 and 5 when observed on human shoulder motion has been called Codman's paradox. Let us show that there is no paradox about it.

We have pointed out that R_{III} (descending) rotation depends on previous two. If we want to return the tip of rotated object to original position, we need to cancel out the effect of rotation that moved it from there, which is R_I. The angle of R_{III} (θ_{III}) must therefore be the same as the angle of R_I in opposite direction ($-\theta_I$). The axis of R_I was rotated by R_{II}, though. This means that the axis of R_{III} (a_{III}) can be obtained by transforming the R_I axis (a_I) by R_{II} rotation:

$$a_{III} = R_{II} \cdot a_I \quad (6)$$

The rotation matrix for rotation of angle θ about axis *a* with coordinates *[x, y, z]* can be found by solving the equation 7:

$$R = I + (\sin\theta)M_a + (1 - \cos\theta)M_a^2 \quad (7)$$

where I is identity matrix and

$$M_a = \begin{bmatrix} 0 & z & -y \\ -z & 0 & x \\ y & -x & 0 \end{bmatrix} \quad (8)$$

Although the formulas 7 and 8 with axis a_{III} from equation 6 and angle $-\theta_I$ will give us the proper R_{III} transformation, it is still not obvious that the crucial equation 5 is valid.

To prove it, we will use another way, how to express the R_{III} rotation. The previous can be used for control.

We have shown, that R_{III} is basically R_I in opposite direction and in R_{II} rotated coordinate system. Let us first rotate the coordinate system back in place and let us denote this backwards rotation R_{II}^{-1} for it can be shown that it truly is the inversion to R_{II}. After that, we can apply the R_I^{-1} as mentioned above. Finally, we must return back to rotated coordinate system by applying the R_{II} again. We get that

$$R_{III} = R_{II} \cdot R_I^{-1} \cdot R_{II}^{-1} \quad (9)$$

Substituting 9 to 5 we get

$$R_{II} \cdot R_I^{-1} \cdot R_{II}^{-1} \cdot R_{II} \cdot R_I = R_{II} \quad (10)$$

Considering the fact, that multiplying the matrix with its inverse, we get identity (unit matrix), we can see that this expression 10 is certainly true.

3 Case of Shoulder

Codman's paradox has been discussed in in connection with human shoulder motion which is quite a special situation. So after describing in general the mathematics of three consecutive rotations that bring the tip of rotated object to original location, we shall now look closer at the Codman's rotations of shoulder.

The coordinate system used to describe shoulder motions is rather specialized to fit the anatomy and the common shoulder motions such as abduction/adduction, flection/extension. Thus one of the coordinate axes coincides with the long axis of human arm in neutral position. The long shape of human arm as the rotated object makes the axial rotation look different from the rotations about other orthogonal axes, although from the mathematics point of view the difference is none.

The whole phenomenon of Codman's rotations tells us one thing about human shoulder joint. *At significant part of its motion range, the behavior of human shoulder joint quite well approximates the ideal spherical joint with 3 degrees of freedom*.

On the other hand, the truth of this statement can be dubious because of complex structure of shoulder joint. This structure's anatomy is well documented in literature Standring (2005). The same applies for various observations of shoulder function Dvir and Berme (1978), although many clinical aspects still remain subject of research. The *shoulder rhythm* Maurel and Thalmann (2000) is a term well suited to express the simultaneous action of all shoulder components. Codman's rotations are performed by such simultaneous action of the shoulder as a whole. Thus the spherical quality demonstrated by phenomenon of Codman's rotations does not really indicate anything about any of shoulder's anatomical components in particular.

3.1 Possible Use of Codman's Rotations

We have shown that the axial rotation that remains after performing the three Codman's rotations is the very same transformation as the second Codman's rotation called *swing* (R_{II} in 2.1). The rotation axis in global coordinate system is the same as well as the angle. The question is, can this fact tell us something about the mobility and motion ranges of the joint?

A possibility of clinical use in motion range diagnostics has been suggested Cheng (2006). Certainly measuring the swing of the arm would be easier than measuring arm's axial rotation. The problem might occur with the complex structure of the shoulder. While we might be interested in glenohumeral joint the clavicle motion could be largely responsible for the range of swing as well as for the ”axially” rotated position after completing the cycle of Codman's rotations. It is not clear to what level of detail reaches the correspondence between arm's swing and axial rotation. Therefore the great attention should yet be given to design of diagnostics procedures utilizing the Codman's rotations.

4 Technical Issues

A simple software tool has been created to demonstrate and understand the idea of Codman's rotations as well as to verify the mathematical reasoning in section 2.1 of this paper. This program was written in Java and VRML standard was used for all 3D visualizations. Java program and VRML based scene were connected using the implementation of EAI (External Authoring Interface) distributed with ParallelGraphics Cortona VRML client.

The rotated object in the testing scene was an L-shaped combination of two boxes, the simplest shape that enables easy visual track of its spatial orientation.

The third Codman's rotation was computed from first and second in both described ways.

5 Conclusion

We have looked on the phenomenon called ”Codman's paradox” from mathematics point of view. We found by theoretical reasoning as well as by practical implementation that there is no real paradox involved. The behavior of human shoulder joint referred to as ”Codman's paradox” merely shows us that the shoulder joint at certain level of detail and within a significant part of its motion range well approximates the ideal spherical joint with 3 degrees of freedom.

However, the correspondence of arm swing and axial rotation might not be entirely without significance. It is possible that clinical methods of joint mobility diagnostics could eventually be based on this knowledge. The research into anatomical aspects of the described shoulder behavior will be necessary in order to invent such methods.

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